Proceedings of the American Academy of Arts and Sciences.

VOL. XLI. NO. 2. — MAY, 1905.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

MEASUREMENT OF VARIOUS THERMAL AND ELECTRICAL EFFECTS, ESPECIALLY THE THOMSON EFFECT, IN SOFT IRON.

BY EDWIN H. HALL, MESSRS. CHURCHILL, CAMPBELL, AND SERVISS.

INVESTIGATIONS ON LIGHT AND HEAT MADE OR PUBLISHED, WHOLLY OR IN PART, WITH APPROPRIATIONS FROM THE RUMFORD FUND.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

MEASUREMENT OF VARIOUS THERMAL AND ELECTRICAL EFFECTS, ESPECIALLY THE THOMSON EFFECT, IN SOFT IRON.

By Edwin H. Hall, Messrs. Churchill, Campbell, and Serviss.*

Presented December 14, 1904. Received March 9, 1905.

Part I: General Account of Methods, Apparatus, and Results.

Let the end E of a bar of metal be maintained at some fixed temperature, T, and let the end E' of the same bar be maintained at a different fixed temperature, T'. If an electric current of given strength is then driven through the bar, the amount of heat generated by this current will depend on the direction of the current, being either greater or less when the current flows from E to E' than when it flows in the opposite direction. This directional relation between an electric current and the heat generated by it is the simplest and most direct aspect of the "Thomson effect."

Let us suppose that we have initially, before the electric current is put on, a uniform ascending gradient of temperature from the end E to the end E', indicated by the straight line S in Figure 1. Let J in the same figure represent the line of temperature gradient which would be established in the bar by a permanent electric current of strength i, if there were no Thomson effect. Let the line A represent the temperature gradient which would actually be maintained in the bar by the permanent current

* Most of the preliminary work of this investigation, making the main thermo-electric couples, testing them, varnishing the iron bars and fixing the thermo-electric couples upon them, assembling the apparatus as a whole and getting it into good working order, was done by Mr. Churchill, who gave much time to the undertaking during the academic years 1902–3 and 1903–4. Unfortunately Mr. Churchill, leaving Cambridge early in September, 1904, was unable to take much part in the main observations, which were made during the fall of that year and the early part of 1905, largely by Mr. L. L. Campbell and Mr. S. B. Serviss of the Harvard Graduate School.

i flowing from E to E'; and let B represent the gradient which would be maintained by an equal current flowing from E' to E. The general disposition of the lines A and B with respect to J in this figure corresponds to the phenomena observed in iron, more heat being generated by a given current in this metal when the flow is from cold to hot than when it is from hot to cold.

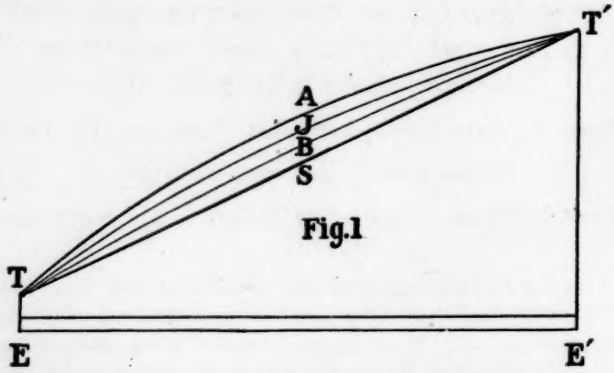


Fig. 1

If we have two similar cylindrical bars of iron, α and β, and send the same current from cold to hot in α and from hot to cold in β, we may have simultaneously gradient A of Figure 1 in bar α and gradient B in bar β. We shall in the discussion that follows assume this condition of things.

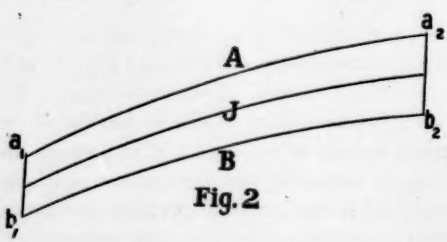


Fig. 2

We shall assume also, for the present, that the bars are so protected that no heat escapes from them laterally.

In Figure 2 we have parts of the curves A and B, the part $a_1\ a_2$ belonging to any part of the bar α and the part $b_1\ b_2$ belonging to the corresponding part of the bar β. Let the length taken on each bar be L cm., and let the area of cross-section of each bar be S sq. cm.

We shall assume that the specific electric resistance of the iron is a linear function of the temperature, and accordingly shall write

$K(1 + \gamma t_a)\ L \div S$ as the resistance of the L cm. of α
and $K(1 + \gamma t_b)L \div S$ as the resistance of the L cm. of β,

K being the specific resistance at 0° C., γ the temperature coefficient of resistance, t_a the mean temperature of the L cm. of α, above 0° C., and t_b the corresponding mean for β.

Let the specific thermal conductivity of the iron be k_{a_1} at the temperature t_{a_1}, k_{a_2} at the temperature t_{a_2}, k_{b_1} at t_{b_1} and k_{b_2} at t_{b_2}.

Let q_a be the Thomson effect heat in $a_1\ a_2$; that is, the amount of heat generated per second in $a_1\ a_2$ in excess of the amount accounted for by Joule's law. Let q_b be the Thomson effect heat in $b_1\ b_2$; that is, the amount by which the heat generated per second in $b_1\ b_2$ falls short of the amount given by Joule's law.

Let the temperature gradient at any point in either bar be called g, with the subscript approximate to the point. The current is to be expressed in amperes, the resistance in ohms, and the heat in calories.

The heat carried out by conduction at a_1 must exceed the heat brought in by conduction at a_2 by the amount generated between a_1 and a_2. Hence

$$S(k_{a_1}g_{a_1} - k_{a_2}g_{a_2}) = q_a + 0.2387\, i^2 K(1 + \gamma t_a) L \div S,$$

or

$$(1) \qquad q_a = S(k_{a_1}g_{a_1} - k_{a_2}g_{a_2}) - 0.2387\, i^2 K(1 + \gamma t_a) L \div S.$$

For the other bar we have

$$(2) \qquad q_b = -S(k_{b_1}g_{b_1} - k_{b_2}g_{b_2}) + 0.2387\, i^2 K(1 + \gamma t_b) L \div S.$$

From (1) and (2) we get

$$(3) \quad q = \frac{q_a + q_b}{2} = \tfrac{1}{2} S[(k_{a_1}g_{a_1} - k_{b_1}g_{b_1}) + (k_{b_2}g_{b_2} - k_{a_2}g_{a_2})] - 0.1194\, i^2 K\gamma (t_a - t_b) L \div S.$$

As the difference of temperature between any part of α and the corresponding part of β is likely to be small, always much less than one degree in the experiments to be described, and as the temperature coefficient of k is small, we may provisionally write in place of (3)

$$(4) \quad q = \tfrac{1}{2} S[k_1(g_{a_1} - g_{b_1}) + k_2(g_{b_2} - g_{a_2})] - 0.1194\, i^2 K\gamma (t_a - t_b) L \div S,$$

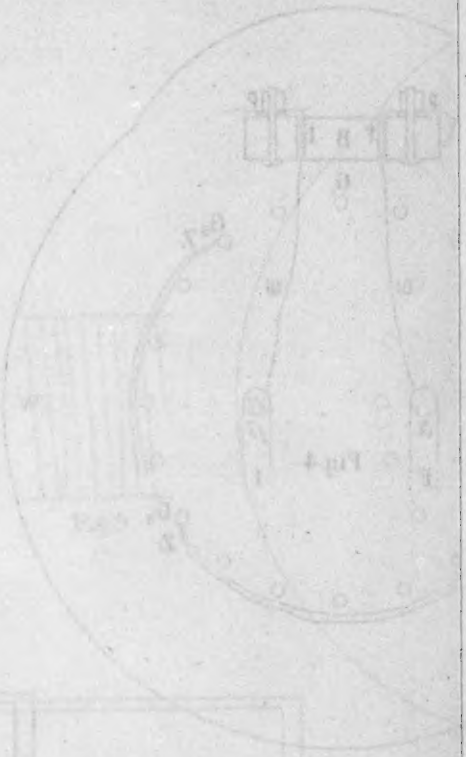
where k_1 is a mean between k_{a_1} and k_{b_1}, while k_2 is a mean between k_{a_2} and k_{b_2}. The magnitude of the error made in the substitution of (4) for (3) will be considered later. It is not very important.

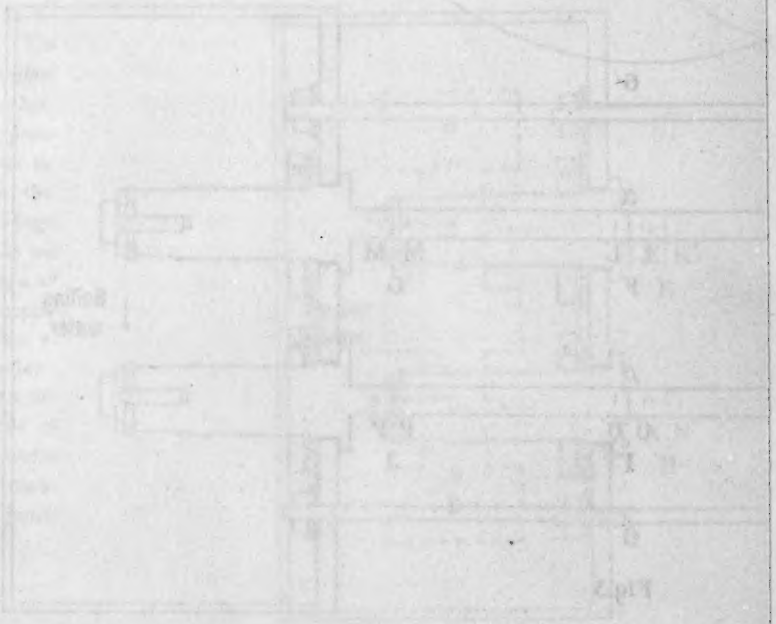
Of the quantities in the second member of (4), S, L, K, i, and γ are easily measured; k_1 and k_2 can, from previous study of the thermal conduc-

tivity of the iron used, be found as soon as the corresponding temperatures are known; the differences of gradient and the differences of temperature at corresponding parts of the two bars are the difficult things. Let us now look at the apparatus by means of which an attempt has been made to determine these latter quantities.

Figure 3 shows a horizontal section through the main part of this apparatus, the scale of the figure being 2 to 5. Each of the bars α and β is 20 cm. long in its narrow cylindrical part, and this part of each bar was ground by Brown and Sharpe to a diameter of 1 cm., with such accuracy that the deviations are difficult to detect and need not be further considered. One enlarged end of each bar is submerged in a mixture of ice and water, which is agitated by a plunging stirrer, P, which makes about forty-five strokes a minute, entering and leaving the water at each stroke and causing about 1.5 cm. fluctuation of the surface level. The other enlarged end of each bar is immersed in boiling water. Each bar is of course electrically insulated from the metal boxes which contain the water. The arrows indicate the general direction of the electric current in what will be called condition B, the current running from cold to hot in the rod which was farthest from the observer, the *back* rod. Half of the time the current ran from cold to hot in the *front* rod, and it was then said to be in condition F. G and G are two of twenty iron bars, each about 6 mm. in diameter, which form a guard-ring around the bars α and β. See also Figure 4. The space between α, β, and the bars G is filled with cotton, rather closely packed. Outside the guard-ring is a wrapping of cotton batting about 5 cm. thick. In spite of the guard-ring and the cotton there is a considerable lateral flow of heat, which must be considered later.

The long cylindrical parts of α and β are covered with an insulating coating of asphaltum varnish well baked on. The points K and K' on α (Figure 3) are the two junctions of a thermo-electric couple consisting of a piece of well-annealed german-silver wire (18 per cent nickel), 0.02 cm. in diameter and 13 cm. long, neatly joined, by means of a small amount of hard solder, to copper wires, also 0.02 cm. in diameter. Both the german-silver wire and the copper wire are wound in a groove, for the most part a spiral groove, made in a second coating of asphaltum varnish while it was yet comparatively soft. The junction K was to be placed 1.5 cm. distant from the neighboring shoulder of α, and the junction K' was to be 2.5 cm. distant from the same shoulder. Accordingly, the groove, already mentioned, into which the wire was to be wound, was made very carefully, by the use of a special tool and a gear-cutting





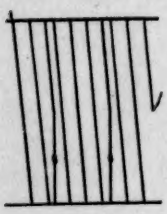


Fig.5

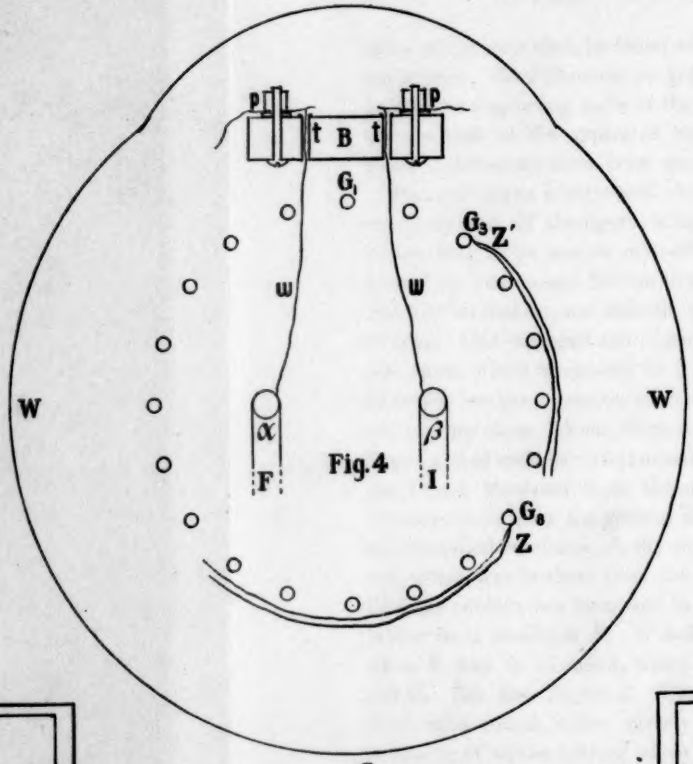


Fig.4

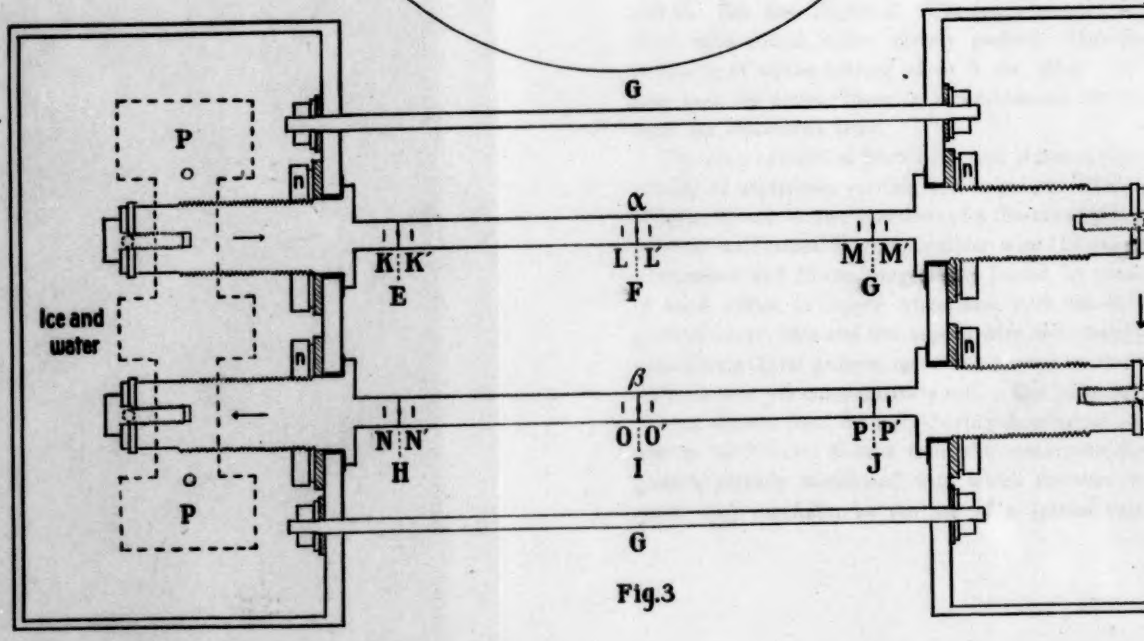


Fig.3

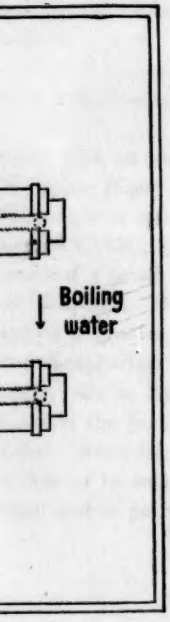
lathe, and was of such shape that, if the layer of varnish could have been removed from the rod and laid out flat, the groove would have looked like Figure 5, the scale of which is 4 to 5. The two dots shown in this figure mark the positions of the junctions K and K'. It will be seen that each of these points lies in a part of the groove which is not of the spiral, a part which is in a plane at right angles with the axis of the bar.

These two plane half circles in which the junctions rest are very nearly 1 cm. apart, perhaps as much as 1.003 cm. In three trial cases the distances between such grooves, after baking, were 1.0021 cm., 1.0018 cm., 1.0032 cm., according to the measurements made by Mr. Churchill by means of a dividing engine. The small lumps of solder at the junctions are probably not quite symmetrical, though they were filed down as much as was considered safe, and therefore the junctions probably do not lie exactly in or over the middle of the grooves. But the conditions of the experiment are so varied, by reversing the current and by reversing the temperature gradient, that the result must be very nearly the same as if each junction were placed accurately in the groove made for it.

The couple $L\,L'$ has its left junction 9.5 cm. distant from the left shoulder of α, and its right junction 9.5 cm. from the right shoulder. The couple $M\,M'$ has M distant 2.5 cm. and M' distant 1.5 cm. from the right-hand shoulder of α. The couples $N\,N'$, $O\,O'$, and $P\,P'$ have corresponding positions on β.

Figure 4 shows a vertical cross-section through the bars near their middle. The small copper wires, w and w, leading up from α and β, pass through glass tubes, t and t, set in the board B, and above are soldered to somewhat larger copper wires, which are gripped near the end by the binding posts p and p, but extend beyond these posts and out of the main apparatus to make connections with the sensitive galvanometer used to measure the thermo-electric currents. Before the copper wires were soldered together they were hooked to each other, and during the soldering the hooks were kept taut, so that after the soldering the two copper surfaces were almost, if not quite, in direct contact. An actual interruption of the copper connection by as much as half a millimeter of solder at this point would probably make serious trouble. The board B was baked after receiving a coating of asphaltum varnish.

Z' is a thermo-electric junction of copper and german-silver laid on the outer side of the bar G_3, with a layer of thin sheet rubber between. Wires leading from this junction pass around the whole guard-ring in a vertical plane, and the german-silver wire joins another copper wire in a second junction, which can be placed in ice-water or in steam. The



object of this arrangement is to give an indication of the temperature at the point where the inner junction lies against the guard-ring bar G_3. For a like purpose the junction Z is placed on the bar G_3, 90° around the guard-ring from G_3.

The line $W\,W$ indicates the boundary of the cotton wrapping of the rods.

E, F, G and H, I, J, in Figure 3, are copper-german-silver junctions, which have, with respect to certain points on α and β, a function like that which the junctions Z and Z' shown in Figure 4 have with respect to points on the guard-ring bars. Thus E, with its corresponding external junction placed in ice-water or in steam, indicates the temperature of a point midway between the junctions K and K'. H does the same for the junctions N and N', etc.

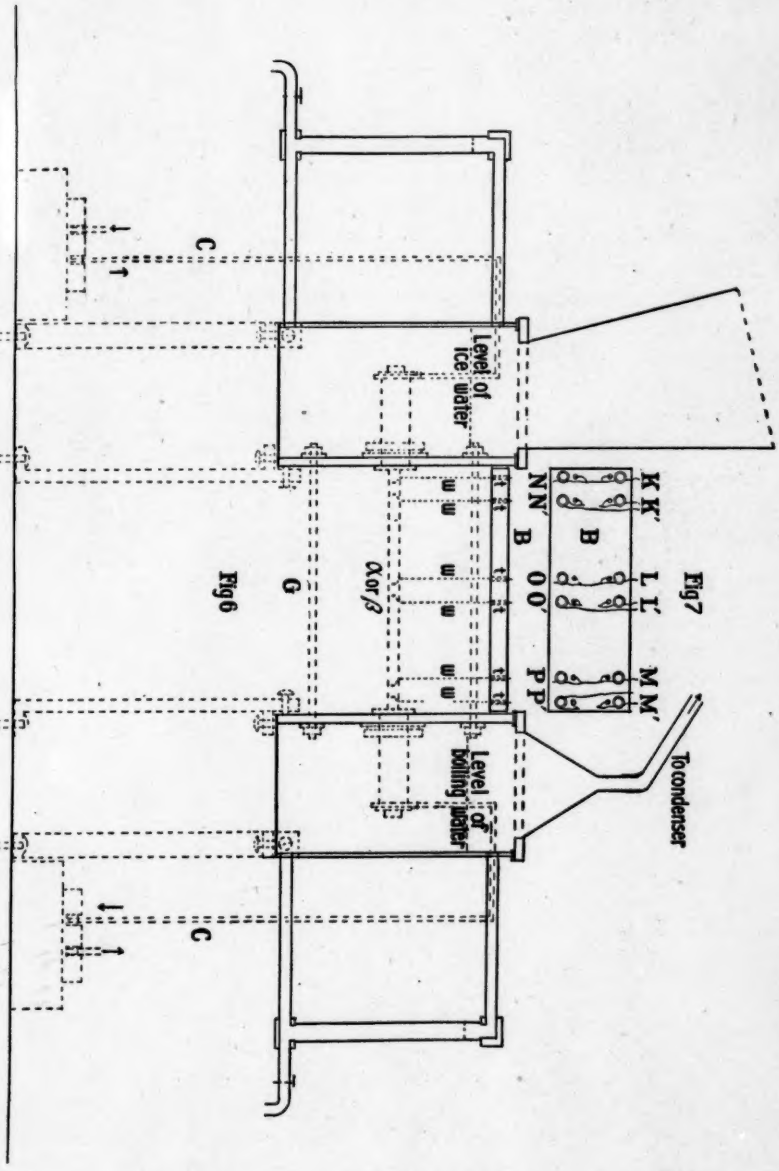
Figure 6 shows a vertical longitudinal section of the apparatus on the scale 1 to 6. C and C are the conductors carrying the main current into and out of the apparatus. This current, about 25 amperes, is measured by an ammeter not here shown.

Figure 7 shows the arrangements on the top of the board B. It is to be noted that each couple on α is joined in opposition to the corresponding couple on β. For example, the wires coming up from the junctions K and N (Figure 3) are soldered to one and the same wire, which leads away from B and may run to the galvanometer, though it is usually disconnected at its outer end. The wires from junctions K' and N', respectively, on the other hand, connect above B with two separate wires, which lead off toward the galvanometer. The three wires thus running from the couples $K\,K'$ and $N\,N'$ make it possible to put into the galvanometer circuit either of these couples alone or both in opposition to each other. The other pairs of couples are similarly connected, L L' with $O\,O'$, and $M\,M'$ with $P\,P'$.

It is unnecessary to depict here the other apparatus used, concerning which some details will be given later.

The general order of procedure on October 22, 1904, a fair sample day, was substantially as follows:

	Time.	
A.M.,	8:05	Water ice-cold in left box, boiling in the other.
	9:00	Main current put on in condition F.
	10:35	Measurement of sensitiveness of galvanometer.
	"	" disturbing thermo-electric effects.
	"	" main current.


C
C
Level of ice water
Level of boiling water
To condenser
K K′
L L′
M M′
N N′
O O′
P P′
B
B
G
α or β
w w w w w w w w
Fig 6
Fig 7

Time.							
10:52	Observations with couples	MM'	and	PP'	opposed.		
	"	"	"	LL'	"	OO'	"
	"	"	"	KK'	"	NN'	"
	"	"	"	LL'	"	OO'	"
11:22	"	"	"	MM'	"	PP'	"
	Measurement of main current.						
	"	"	disturbing effects.				
11:35	"	"	sensitiveness.				
	"	"	main current.				
11:45	Observations with couples	MM'	and	PP'	opposed.		
	"	"	"	LL'	"	OO'	"
	"	"	"	KK'	"	NN'	"
	"	"	"	LL'	"	OO'	"
P. M., 12:18	"	"	"	MM'	"	PP'	"
	Measurement of main current.						
	"	"	disturbing effects.				
12:35	"	"	sensitiveness.				
	"	"	main current.				
12:54	Observations with couples	MM'	and	PP'	opposed.		
	"	"	"	LL'	"	OO'	"
	"	"	"	KK'	"	NN'	"
	"	"	"	LL'	"	OO'	"
1:30	"	"	"	MM'	"	PP'	"
	Measurement of main current.						
1:36	*Reversal*	"	"	"	to condition B.		
	Measurement of disturbing effects.						
1:45	"	"	sensitiveness.				
	Wait for effects of reversal to occur.						
2:26	Measurement of sensitiveness.						
	"	"	disturbing effects.				
	"	"	main current.				
2:42	Observations with couples	MM'	and	PP'	opposed.		
	"	"	"	LL'	"	OO'	"
	And so on till the observations made with main current in condition B correspond in number and variety to						
5:50	those made with current in condition F.						
	Throughout the day various incidental observations, of the barometer, the temperature of the galvanometer, etc., were occasionally made.						

It will be observed that such a day's work includes six complete cycles, or runs, through all the pairs of corresponding couples from MP to KN and back, three runs with the main current in condition F, and three with this current in condition B. If we let

A = strength of main current in amperes,
t_w = temperature of boiling water at mid depth,
E_c = net e. m. f., in volts, of the opposed cold couples,
E_m = " " " middle " ,
E_h = " " " hot " ,

the sign + indicating that the couple on the warmer bar, the bar in which the current runs from cold to hot, prevails, and *vice versa*, we can express the results of the observations of October 22 as follows:

	A	E_c	E_m	E_h	t_w
Condition F	25.06	-257×10^{-9}	$+585 \times 10^{9}$	-120×10^{-9}	100°
Condition B	25.02	$+1450 \times 10^{-9}$	-314×10^{9}	-1185×10^{-9}	100°
Means . .	25.04	$+597 \times 10^{-9}$	$+136 \times 10^{9}$	-653×10^{-9}	100°

It is evident that observations made without reversal of the main current would be useless. This fact is probably due in part to the slight misplacements of the junctions which have already been suggested and in part to certain accidental causes which may vary somewhat with the conditions of the experiment.

To find an approximate value for the difference of temperature gradient which yields any one of the E's given above, we have only to divide this E by the "thermo-electric height" of copper and german-silver at the mean temperature of the two opposed couples which have this net E. Thus we get from the observations of October 22, if Δ stands for difference of temperature gradient in degrees per centimeter, the + sign indicating that the gradient is steeper in the warmer bar, and *vice versa*,

Δ_c	Δ_m	Δ_h
+ 0.0352	+ 0.0072	− 0.0316

It so happens that on this day the value found for Δ_h was numerically less than the value found for Δ_c; but the opposite condition was

the common one, as the following table, which gives the results of all the fifteen days of successful observation of the main phenomena, will show:

Date.	Hot End.	A	Δ_c	Δ_m	Δ_h	t_w
1904.						
Aug. 31	*K–N*	24.98	0.0344	0.0070	− 0.0348	100.6°
Sept. 7	*M–P*	25.02	0.0346	0.0052	− 0.0341	100.4°
" 8	"	25.18	0.0350	0 0047	− 0.0355	100.3°
" 9	"	25.06	0.0367	0.0080	− 0.0347	100.5°
" 21	"	25.24	0.0320	0.0085	− 0.0385	100.5°
" 22	"	25.14	0.0346	0.0071	− 0.0349	100 8°
" 26	*K–N*	25.06	0.0346	0.0072	− 0.0390	100.1°
" 28	"	25.20	0.0337	0.0064	− 0.0386	100.6°
" 29	"	25.30	0.0380	0.0110	− 0.0335	100.2°
" 30	"	25.22	0.0315	0.0042	− 0.0397	99.7°
Oct. 12	"	25.44	0.0345	0.0091	− 0.0351	100.6°
" 22	*M–P*	25.04	0.0352	0.0072	− 0.0316	100.0°
" 25	"	25.12	0.0326	0.0047	− 0.0344	100.5°
" 27	"	25.12	0.0328	0.0079	− 0.0357	100.3°
" 28	"	25.10	0.0357	0.0067	− 0.0368	100.7°
Means		25.15	0.0344	0.0070	− 0.0359	100.4°

There is a certain inaccuracy in the method, already described, by which the Δ's are obtained from the E's. Each E has been treated as if each of the two opposing couples producing it had the same mean temperature. But observations to be described later, made by means of the junctions E, F, etc., shown in Figure 3, indicate that the temperature of the warmer bar exceeds that of the cooler by 0.055° at the mid section of the cool couples, by 0.265° at the mid section of the middle couples, and by 0.145° at the mid section of the hot couples. These differences would, because of the variation of "thermo-electric height" of copper and german-silver with temperature, give appreciable values of E if there were

no differences of gradient in the opposing couples. Applying to the mean Δ's given above the corrections made necessary by this consideration, we get, as the improved Δ's,

Δ_c	Δ_m	Δ_h
+ 0.0337	+ 0.0040	— 0.0374

Dividing each of these quantities by 1.002, which according to a previous statement may be taken as the average distance between the two junctions of a single couple, we get

Δ_c	Δ_m	Δ_h
0.03363	0.00400	— 0.03733

The mean temperatures found for the mid points of the couples $K K'$ $N N'$, etc., by use of the junctions E, H, etc., are

t_c	t_m	t_h
13.0°	51.1°	90.3°

According to a previous investigation,* the value of the thermal conductivity of the iron here studied is 0.1528 at 28° C., with a mean temperature coefficient — 0.0003 for the range from 28° to 58°. A very recent investigation (see p. 52), made with the same apparatus that has been described in this paper, gives for the mean temperature coefficient of thermal conductivity from 13° C. to 87° C. the value — 0.0007. Using this later value and taking k as 0.1528 at 28°, we get

k_c	k_m	k_h
0.1543	0.1493	0.1461

From the difference of temperature of the two bars, as already given, at the sections c, m, and h respectively, we can make the somewhat uncertain estimate that the difference $(t_a - t_b)$, occurring in equation (4), is, between the sections c and h, 0.21°.

Recent measurements, by Mr. Campbell, of the electrical conductivity of the iron here used give for K, in equation (4), the value 11365×10^{-9} ohms and show that γ is approximately 0.005.

Putting, in equation (4),

* Physical Review, May–June, 1900, p. 305.

$$S = 0.785,$$
$$L = 16,$$
$$k_1 = k_c \text{ above} = 0.1543,$$
$$k_2 = k_h \quad " \quad = 0.1461,$$
$$g_{a_1} - g_{b_1} = \Delta_c \text{ last given} = 0.03363,$$
$$g_{b_2} - g_{a_2} = -\Delta_h \quad " \quad " \quad = 0.03733,$$
$$i = 25.15,$$
$$K = 11365 \times 10^{-9},$$
$$\gamma = 0.005,$$
$$t_a - t_b = 0.21,$$

we get, as the first approximation to the value of the Thomson effect heat,

$$(5) \qquad q = 0.002037 + 0.002142 - 0.000018.$$

In this evaluation of q we have taken full account of the temperature gradients; but we have, in using a mean k_1 for the cold couples and a mean k_2 for the hot couples, ignored the fact that the thermal conductivity is slightly less at any point in the warmer bar than at the corresponding point in the cooler bar. In fact, if there were no difference of gradient at the cold couples, the difference of temperature, about 0.055°, between the bars at the section c would, with the temperature gradient 4.6, which is, approximately, the general gradient along each bar, send about 0.000020 calorie per second more along the cooler bar than along the other, and accordingly the first term in the second member of equation (5) should be diminished by half this amount. At the h section the excess of temperature of the warmer bar is apparently about 0.145°, and accordingly, if there were no difference of gradient in the two bars at this section, the cooler would here transmit about 0.000056 calorie per second more than the warmer. Therefore the second term of the second member of equation (5) should be increased by half this amount The result is,

$$(6) \qquad q = 0.002027 + 0.002170 - 0.000018 = 0.004179.$$

This would be the final value of q, if there were no correction for loss of heat by lateral flow from the bars α and β. Unfortunately such a correction is necessary and is large. It is estimated by the following method: When both ends of both bars have been for several hours in boiling water, the couples, KK', NN', MM', and PP' indicate a perma-

nent temperature gradient of about 0.1° per centimeter from the ends toward the middle of the bars. Under the same conditions, observations made with the junctions which are attached to the guard-ring bars (see Figure 4) indicate that the mean temperature at the outer surface of the guard-ring, for the 16 cm. corresponding to the distance between sections KN and MP of the main bars, is approximately 4.5° below the temperature of the boiling water in which the ends of α and β are immersed. Taking the mean temperature of the main bars in this case for the stretch between section KN and section MP as 0.3° below the temperature of the water, which in view of all the observations seems a reasonable estimate, we get 4.2° as the mean difference of temperature maintaining lateral flow of heat, a flow sufficient to maintain a gradient of 0.106° per centimeter along each bar toward the middle at both of the end sections. The amount of heat thus transmitted per second is about 0.0242 calorie from each bar.

The lateral loss from each bar when one end is hot and the other cold is probably considerably less than half this amount, as the difference of temperature between corresponding points on the main bars and the guard-ring bars is in this case apparently less than 2°; but even this diminished flow is greater than the whole Thomson effect heat, and would be very serious indeed if it were not very nearly the same from both bars. The warmer bar being, according to the estimate already made for $(t_a - t_b)$, about 0.21 warmer than the other, the excess of lateral flow from the warmer bar may be estimated as $0.0242 \times (0.21 \div 4.2)$ calorie per second. This is 0.001209 calorie per second, and the value found for q in equation (6) should be increased by one half this amount, which will give as our final value

(7) $$q = 0.00478.*$$

To find σ, the mean amount of heat produced per second by sending 10 amperes, 1 c. g. s. absolute unit of current, through a temperature interval of 1° downward in the region between 13.0° and 90.3°, we have

$$\sigma = -q \div 2.515 \times (90.3 - 13.0) = -0.0000246.$$

If we assume that the heat generated or absorbed per second by a given current flowing through an interval of 1° is proportional to the absolute temperature of the metal at the point considered, — that is, if we assume that the line representing iron on the ordinary thermo-electric diagram

* The value of q would be 0.00483, if -0.0003 had been taken as the temperature coefficient of thermal conductivity. See p. 33.

should be a straight line within the range of temperature here considered, we get as the Thomson effect coefficient,

$$\nu = \sigma \div (273 + \tfrac{1}{2}(90.3 + 13.0)) = -757 \times 10^{-10}.$$

Attempts to determine the Thomson effect directly in precise units have, apparently, been very few. Le Roux in 1867 * gave the result of a rough absolute measurement of this effect in a certain alloy of bismuth and antimony, called "bismuth of Becquerel." Battelli published in 1886 † the results of measurements made by himself with various metals, one of which was iron. Mr. R. O. King published in 1898 ‡ the method and results of a study in absolute measure of the Thomson effect in copper, his work having been done at the Jefferson Physical Laboratory of Harvard University, though the general plan of his investigation came from Professor Callendar at Montreal.

Battelli, whose method seems not capable of very great accuracy, though it was probably a great improvement over that of Le Roux, got data which yield the following values of ν for iron:

$$-283 \times 10^{-10} \text{ at } 53°\text{C.}$$
$$-319 \times 10^{-10} \text{ " } 108°\text{C.}$$
$$-331 \times 10^{-10} \text{ " } 243°\text{C.}$$
$$-369 \times 10^{-10} \text{ " } 308°\text{C.}$$

He concludes from his own experiments: "1. That in all the substances studied, cadmium, iron, antimony, bismuth, *pakfong* [german-silver?], an alloy of 10 Bi + 1Sb by weight, the Thomson effect is proportional to the intensity of the current. 2. That in all these substances, with the exception of iron, the Thomson effect [the σ, not the ν, of this paper] is proportional to the absolute temperature."

The value of ν obtained from Battelli's data for the temperature 53°, which is very near the mean temperature, 51.7°, of the range used in the experiments described in this paper, is about 0.37 of the value here found for ν. Battelli gives no description whatever of the quality of the iron which he studied. It was "*ferro*." His apparent discovery that the ν for iron increases with rise of temperature, which requires that the line representing iron in the ordinary thermo-electric diagram shall be a curve growing steeper at high temperatures, has probably not been generally

* Ann. de Chimie et de Physique, 4me série, t. X, p. 201.

† Atta della Reale Accademia delle Scienze di Torino, **22**, 48, 539.

‡ These Proceedings, **33**, 19, June, 1898.

accepted; though the fact that he does not find a change of slope for the lines of the other substances examined by him may well make us hesitate to reject, as based on experimental errors, his conclusion concerning iron.

The data of this paper enable us to make a separate calculation of the Thomson effect for the stretch from section c to section m and for the stretch from m to h; and this calculation is perhaps worth making, though the results will naturally be less reliable than the one result for the whole stretch from c to h.

Putting, in equation (4),

$$\begin{aligned} S &= 0.785 \\ L &= 8.0 \\ k_1 = k_c &= 0.1543 \\ k_2 = k_m &= 0.1493 \\ g_{a_1} - g_{b_1} = \Delta_c &= 0.3363 \\ g_{b_2} - g_{a_2} = -\Delta_m &= -0.00400 \\ i &= 25.15 \\ K &= 11365 \times 10^{-9} \\ \gamma &= 0.005 \\ t_a - t_b &= 0.21, \end{aligned}$$

we get, as the first approximation to the Thomson effect heat in the stretch from section c to section m,

$$(5') \qquad q' = 0.002037 - 0.000235 - 0.000010.$$

Correcting for the effect of the higher temperature of the warmer bar in diminishing its thermal conductivity, we get

$$(6') \qquad q' = 0.002027 - 0.000186 - 0.000010 = 0.001831;$$

with allowance for lateral leakage we get

$$(7') \qquad q' = 0.002090.$$

To find q'', the Thomson effect heat for the stretch m to h, we have

$$(7'') \qquad q'' = q - q' = 0.00478 - 0.00209 = 0.00269.$$

The value of the Thomson effect coefficient for the stretch c to m would, according to the data pertaining to this stretch alone, be

$$\nu' = -q' \div (2.515 \times 38.1 \times 305) = -715 \times 10^{-10}.$$

The value of this coefficient found from the data belonging to the stretch m to h alone is

$$\nu'' = -q'' \div (25.15 + 39.2 \times 344) = -793 \times 10^{-10}.$$

The difference between ν' and ν'', about 10 per cent, is too large to be readily accounted for by experimental errors. Taken with the evidence found by Battelli, it may perhaps be regarded as raising a presumption in favor of the opinion that ν does increase, numerically, with rise of temperature in iron.* On the other hand, the common practice of drawing a straight line for iron, at low temperatures, on the thermo-electric diagram is not based on pure imagination or on mere theory. For example, experiments made on a thermo-electric couple, consisting of the iron here studied combined with copper, indicated † that the "thermo-electric height" of this couple diminished at a nearly uniform rate with rise of temperature, which may seem at first sight to require either lines very nearly straight for both iron and copper on the thermo-electric diagram, or lines having very nearly equal changes of curvature with the region of temperature used, 26.6° to 71.1°. Inspection of the ordinary diagram will show, however, that in this range of temperature a considerable relative change in the inclination of either line would make a much smaller relative change in the thermo-electric height, which is represented by the vertical distance between the two lines. Such change as there was in the rate of change of this height with rise of temperature was in the right direction to be accounted for by an increase in the steepness of the slope of either the iron line or the copper line. The values found for the "thermo-electric height" of the copper-iron couple were:

at 26.6° C.	1028×10^{-8} volt
" 41.3°	980×10^{-8} volt
" 54.5°	938×10^{-8} volt
" 71.1°	870×10^{-8} volt

* Tait, Trans. Roy. Soc. Edin., **27** (1872–76), 137, found that at high temperatures the iron line has at least two curvatures, each sufficient to change the sign of its inclination. Tait made no absolute determinations of the Thomson effect, but based the inclination of the lines of his diagram on the assumption (supported by the experiments of Le Roux) that this effect is null in lead. Battelli (Atti della R. Accademia dei Lincei, serie 4, vol. 3, 1887, p. 212) found data which give 4.3×10^{-10} as the value of ν in lead.

† These Proceedings, **36**, 130, August, 1900.

To establish a uniform gradient the intermediate values should have been

at 41.3°	975×10^{-8}
" 54.5°	929×10^{-8}

With these data, and with the value of ν found in this paper for iron, it is possible to go through the form, at least, of calculating the value of ν for copper. The process is as follows:

E. m. f. of copper-iron couple with junctions at 26.6° C. and 71.1° C., respectively, is

$$E = \frac{1028 + 870}{2} \times 10^{-8} \times (71.1 - 26.6) \text{ volt} = 422 \times 10^{-6} \text{ volt.}$$

Heat, calories, which would be absorbed by this net e. m. f. in one second by 10 amperes is

$$q_n = 422 \times 10^{-6} \times 10 \times 0.2387 = 0.00101.$$

Heat, calories, which would be absorbed at the hot junction in one second by 10 amperes is

$$q_h = 870 \times 10^{-8} \times (273 + 71.1) \times 10 \times 0.2387 = 0.00715.$$

Heat which would be given out at the cold junction by the same current in the same time is

$$q_c = 1028 \times 10^{-8} \times (273 + 26.6) \times 10 \times 0.2387 = 0.00735.$$

Hence the heat which would be absorbed in the Thomson effect in both metals by the same current in the same time is

$$q_T = 0.00101 - (0.00715 - 0.00735) = 0.00121.$$

Letting ν_c and ν_i stand for the Thomson coefficients in copper and iron respectively, we have

$$\nu_c - \nu_i = q_T \div (71.1 - 26.6) \left(273 + \frac{26.6 + 71.1}{2}\right) = 844 \times 10^{-10}.$$

Taking $\nu_i = -757 \times 10^{-10}$, the mean value found in this paper, we get

$$\nu_c = 844 \times 10^{-10} - 757 \times 10^{-10} = 87 \times 10^{-10}.$$

King, in the paper already referred to, gave evidence from which he concluded that the Thomson effect σ for copper diminishes with rise of temperature. The ν would of course diminish still more rapidly. He

worked out values of σ for several temperatures, with various assumptions as to the thermal conductivity of his copper and the radiation from its surface. The smallest value of ν which comes from any of his estimates of σ is 103×10^{-10} at 208°; the largest value, 174×10^{-10} at 122°.

The fact that King's smallest value of ν is considerably larger than the value of ν_c deduced from the ν_i found in this paper for iron may indicate that the ν_i as here found is somewhat too large. It is to be observed that a reduction of a little more than two per cent in the numerical value used for ν_i would bring the value of ν_c up to the minimum value found from King's data. It is quite possible that the value 757×10^{-10} found for ν_i is several per cent in error.

We are at present making preparations for a study of the properties of our iron between 100° C. and 200° C.

It may be advisable to make a direct determination of the Thomson effect in copper and various other metals with apparatus like that described in this paper, but the difficulty of a direct study of copper would be very considerable.

The properties of the very soft iron studied in this paper, so far as they are quantitatively known, are summarized below:

Composition: iron, 99.93 %; carbon, 0.059%.

Density: about 7.785 at 0° C.

Thermal conductivity, k: 0.1528 at 28.2° C.,
with a temperature coefficient 0.0003 (?) from 28° to 58°,
" " " " 0.0007 (?) " 13° " 87°,
the latter value being the more reliable.

Electric resistance, absolute: 11365 at 0° C.,
with a mean temperature coefficient 0.00519 from 0° to 100°.

"Thermo-electric height" with copper:

approximately
1028×10^{-8} volt at 26.6° C.,
980 " " " 41.3°,
936 " " " 54.5°,
870 " " " 71.1°,

Thomson effect coefficient, ν:

approximately
-757×10^{-10} mean, from 13° to 90°,
-715 " " " " " 51°,
-793 " " " 51° " 90°.

Part II: Details.

Preparation of the Main Bars.

The two main soft-iron bars, α and β in Figure 3, were cut from the same piece of metal, and each was marked at one end. The two marked ends came from the same end of the original bar, and were placed at the same end of the apparatus for the main experiment.

For an insulating layer on the narrow part of these bars, paper baked on with asphaltum varnish was at first tried; but it did not adhere well, and the varnish alone was finally used, baked on in an electric oven.

An attempt was made to *cut* a groove for the fine wires of the thermo-electric couples in a second layer of asphaltum, baked on at 100° C. over a first layer baked on at 200° C.; but this failed, the varnish chipping off under the action of the cutting tool.

These experiments, and various others which need not be described, were made with trial bars of iron, not with the bars α and β, as the complete removal of the varnish from a rod in case of failure is a troublesome operation. The method finally adopted and used with bars α and β was substantially as follows: A thin layer of asphaltum varnish was laid on and, before baking, was allowed to harden at the ordinary temperature of the laboratory for three hours. This partial slow drying was to prevent the varnish from running and distributing itself unevenly during the baking, which lasted four or five hours at 200° C. Then a second thin coating of varnish was put on and allowed to dry at the temperature of the laboratory for two days; then a third thin layer, which also was left to dry during two days. In the unbaked double layer thus provided a groove (see Figure 5) was made by means of a small, sharp-edged, brass disk, held in such a way as to allow revolution with little or no lateral motion and applied with moderate pressure to the rod, which was carried beneath the disk in a screw-like motion by the action of a gear-cutting lathe. After the grooves were thus made the bars were baked about four hours at 200° C. The varnish ran a little into the grooves during this heating, but not enough to cause serious difficulties.

The construction of the six thermo-electric couples which are placed on bars α and β has been described in connection with Figure 5. As a cement to be used temporarily during the placing of the couples in the grooves prepared to receive them, a mixture of beeswax and honey in equal parts was found satisfactory. After this placing, the windings of wire were covered with asphaltum, which was allowed to dry over night,

after which the distance between the junctions of each couple was tested by means of calipers; and one or two junctions were moved a trifle in the still yielding varnish in order to make this distance, as nearly as might be, 1 cm. The windings were then recoated with asphaltum and baked at 100° for some hours, then cooled and re-examined, then coated once more at the points where the copper wires leave the rods, then finally baked for six hours at 200° C.

No grooves were made to receive the supplementary thermo-electric junctions (see F and I, Figure 4), which were put on to give the temperatures of the midway points of the various couples now fixed, according to the method just given, on the bars; for it was not necessary to place these supplementary junctions with any great exactness. They were at first put on with a thick layer of asphaltum, which was allowed to dry for some ten days before baking; but when baking was attempted the heat so softened this coating that it was necessary to start anew. Success in fastening these junctions was finally attained by keeping the new coating of varnish in which they were imbedded at 50° for several hours before the final baking. The worst placed of these junctions at the last was the middle one on bar *a*, which was about half a millimeter too far toward the $K K'$ end of the bar. Such a displacement must have produced certain temporary dissymmetries in the observations, but, with the various reversals which were practised, cannot have had any serious effect on the final result.

Calibration of Thermo-electric Couples.

With the same kinds of wire, and the same methods of annealing and soldering, Mr. Churchill made, for calibration use, four more thermo-electric couples, *A*, *B*, *C*, and *D*. The german-silver wire in each of these was about 1.6 m. long. Near each end it ran through a glass tube about 40 cm. long, to keep it from accidental contact with the copper wire, which was lashed to the outside of the tube. For a calibration test each junction, with a considerable part of the glass tube above it, was placed in a mixture of ice and water or in water near the temperature of the room, or in the vapor of some boiling liquid, as the bulb and stem of a thermometer under test would be placed. The temperatures, except those of melting ice and boiling water, were determined by means of one or the other of a pair of Baudin thermometers, which Mr. Churchill had carefully compared with a Tonnelot corrected to the hydrogen scale by the Bureau International des Poids et Mesures.

Several temperatures between 0° and 100° were used, the room temperature, about 18°, and the boiling-points of ether, bisulphide of carbon, chloroform, ethylacetate, and benzene, respectively. Using all four of the couples *A*, *B*, *C*, and *D* in each test, Mr. Churchill found substantially the following mean values of the "thermo-electric height" for the temperatures given alongside:

Temp.	Volt.
10° C.	1686×10^{-8}
20°	1734×10^{-8}
30°	1782×10^{-8}
40°	1835×10^{-8}
50°	1878×10^{-8}
60°	1925×10^{-8}
70°	1972×10^{-8}
80°	2022×10^{-8}
90°	2073×10^{-8}

These numbers indicate a very nearly constant rate of increase of the sensitiveness of a copper-german-silver couple with rise of mean temperature on the hydrogen scale.

As Mr. Churchill left in Cambridge no detailed account of this calibration work, it seemed advisable to repeat some of it as a check. Accordingly on November 5, 1904, couples *A*, *B*, *C*, and *D* were tested again, each with one junction in a mixture of ice and water, and the other junction in steam, each junction being subjected to both conditions in the course of the trial. The result was somewhat alarming, as it indicated for the sensitiveness of the couples a value about 6 per cent less than Mr. Churchill had found; and further study of the matter seemed necessary.

In the experiment of November 5, as in those of Mr. Churchill, the junctions had been exposed naked to the water and to the steam, a test made with one junction in water and one in alcohol, the two being at or very near the same temperature, having appeared to show that no ill effects need be feared from the chemical action or the electrical conductivity of the water. After November 5, however, this practice of direct exposure of the junctions was abandoned, and in all subsequent calibration tests the glass tubes carrying the junctions were placed in glass tubes, about 38 cm. long and 0.8 cm. in diameter of bore, containing paraffine oil; and these outer tubes were exposed to the water and the vapors used. The

thermometers also were placed in similar oil tubes, lest the direct action on them of the water and the vapors might affect them more rapidly than the junctions would be affected through the oil.

With this arrangement another test of couples *A*, *B*, *C*, and *D* was made November 18, with ice and water and steam, as on November 5. This gave a result about 1.5 per cent less than the one found by Mr. Churchill for the same range of temperature.

On December 2, a test of the same couples was made with a range of temperature from 0° to 56.9° (vapor of boiling acetone); and the result was a value about 2 per cent less than Mr. Churchill had found for the same range.

About the first of December, 1904, four new calibration couples were made from the same kinds of wire that had been used before. In the present case the german-silver wire was annealed, in pieces about 1.2 m. long, by sending through each piece a current of about three amperes (requiring about thirty-five volts) until the wire glowed brightly in a darkened room. Each piece stretched a good deal during the heating and was allowed to hang as a free catenary in the air; but on cooling it returned, apparently, to its original length. Soft solder was used in making these new couples, which will be called *A'*, *B'*, *C'*, and *D'*, respectively. These couples were mounted and were tested in oil, just as the couples *A*, *B*, *C*, and *D* had been mounted and tested at the last. On December 5, experiments with *A'*, *B'*, *C'* and *D'* gave a result about 1.2 per cent greater for the range from 0° to 56.7° than Mr. Churchill had found for the same range. On December 6, with a range from 0° to 100°, experiments with the same junctions gave a result about 1.7 per cent greater than Mr. Churchill's for the same range.

Taken all together, the tests now described, made with couples *A*, *B*, *C*, *D*, *A'*, *B'*, *C'*, and *D'*, in oil, confirmed the values found by Mr. Churchill; and accordingly his values were used in reducing the observations made with the couples which he had made and had attached to the main iron bars.

Perhaps more attention has been given to this matter than the immediate object in view, the determination of the Thomson effect, required. For, as the couples were used, an error of, for example, 1 per cent in the reduction factor applied to their indications would make, to be sure, an error of one per cent in the estimation of the amount of heat generated, but it would make also an error of one per cent in the same direction in the estimation of the interval of temperature between the junctions on the bars. These two errors would balance each other, except in this par-

ticular, that they would leave an error of about one sixth of one per cent in the estimation of the mean temperature of the bars in which the Thomson effect occurs.

Further and more exacting uses of these couples, or others similar, are in view, however; and the study of their behavior will be continued. See p. 54.

Resistances.

The resistances used in this investigation were, for the most part, in the form of detached coils of manganine, made and tested by the methods which Professor P. O. Peirce has established at the Jefferson Laboratory. The greatest proportional error found in any of these coils when they were tested at the end of the work, in December, 1904, was about one third of one per cent in a certain 10 ohm coil, which proved not to be made of the best wire. Error from the use of this coil was probably not more than one seventh of one per cent in any important place, and this error has been disregarded in the calculations.

The resistances of the thermo-electric couples and of the connecting wires of the various circuits were usually measured by means of a well-tested Wheatstone bridge, in box form.

Current Measurers.

On and after August 31, 1904, the instrument used for the measurement of thermo-electric currents was a three-cased *Panzer-galvanometer* of the du Bois and Rubens design. This was entirely satisfactory in regard to sensitiveness and freedom from external magnetic disturbances; but on the other hand its sensitiveness, when great, was subject to large and quick changes, sometimes amounting to several per cent in a single hour, and accordingly it was necessary to make frequent measurements of its reduction factor. For this purpose a potentiometer method was used, with a cadmium cell as a standard of electromotive force and with a number of well-tested resistances. Compared, about the middle of November, 1904, with a number of other cells in the care of Professor B. O. Peirce, the cell here used appeared to have an electromotive force of 1.0178 volt. Tested more carefully in March, 1905, under the direct supervision of Professor Peirce, it showed, near 16° C., an electromotive of 1.0184 volt.

The galvanometer coils were marked by the manufacturer as having 5.5 ohms and 5.3 ohms resistance, respectively, at 20° C., and this certificate was found to be correct.

The heavier of the two suspension systems was used, and no attempt was made to attain the highest sensitiveness. With a scale distance not far from 2.5 meters this galvanometer usually gave about 1 cm. deflection for a current of 10^{-8} amperes, without reversal.

The current, about 25 amperes, which was sent through the iron bars, was measured by means of a shunted ammeter, which, tested * in January, 1905, appeared to have, in the part of the scale used in the experiments of this paper, an error less than one part in one thousand. Tested two or three months later at the Jefferson Laboratory, on the basis of a Crompton standard resistance and a cadmium cell, it read about one half of one per cent low in the same part of the scale. It is assumed in this paper that the instrument gave correct readings as it was used in 1904.

Disturbing Currents.

Much care was taken to avoid accidental thermo-electric currents. For example, unnecessarily thick layers of solder at joints between pieces of copper were studiously avoided. It was not practicable, however, to get rid of mercury contacts, and slight differences of temperature near such contacts sometimes gave currents sufficiently great to require especial attention. The mere operation of a commutator having pieces of copper dipping into little mercury wells, the reversals occurring rather oftener than once a minute, appeared to produce a cumulative disturbing effect, the slow increase going on for hours. The various reversals of other connections which were regularly practised had a tendency to eliminate the effect of such disturbances from the final result; but in some cases it was necessary to measure the illegitimate current regularly and make allowance for it.

Deflections of the galvanometer were sometimes noticed which appeared to be due to slight currents entering the galvanometer circuit from some external non-metallic path. For example, when connections were made for getting the reduction factor of the galvanometer, if the circuit of this instrument remained connected at one point with the thermo-electric couple wound on the *cold* end of either iron bar, α or β, and if at the same time the commutator leading the main current into the iron bars was closed, the galvanometer showed a considerable illegitimate current, the direction and magnitude of which were not changed by reversing the commutator of the twenty-five ampere current, though the

* By the kindness of Professor Laws at the Massachusetts Institute of Technology.

disturbance disappeared when this commutator was opened. The storage battery furnishing the main current was in the basement, and the galvanometer rested on a brick pier which rose from this basement. The disturbance was apparently not due to the storage battery, as its direction was independent of the direction of the current entering the Thomson effect bars from this battery. It probably was due to battery action between water and metals at the cold end of the apparatus, causing a slight current to creep along the varnished surface of the board B (Figure 6), this surface being slightly dampened by precipitation of water from the air, thence along a metal path through the galvanometer, thence by way of the pier surface, or some other damp non-metallic path, to the basement, thence by way of the main current leads to the Thomson apparatus again. When connection was made with the wires leading from the *hot* end of the board *B*, no disturbing effect due to this connection could be made out with certainty. There was, of course, no need of having any metallic connection between the Thomson effect apparatus and the galvanometer circuit during the measurement of the reduction factor; but for convenience the one-point connection described above was frequently left during the early part of the work, before the possibility of trouble from it was thoroughly realized. Fortunately this connection was made, as a rule, with the hot end of the Thomson effect apparatus, so that when the danger was discovered it was necessary to reject on this score the work of one or two days only.

Moisture at the ends of the board *B* was, indeed, a frequent and serious cause of trouble, especially at first. Sometimes it came from leakage past the rubber gasket which separated each vessel from its cover or top. To avoid this leakage the level of the water in each vessel was, after a time, kept below the level of the gasket, and care was taken not to let the boiling become so violent as to clog the condenser return pipe and raise the pressure of steam in the boiler. At the cold end, especially in warm weather, profuse precipitation of water from the air was capable of drenching the cotton lying on the board. To prevent this, the top of the cold vessel was swathed in cotton to a depth of about 2 cm., and outside this cotton was placed a covering of thin sheet rubber.

Much trouble would have been saved by placing the board *B* considerably higher, and by having the end posts upon it placed nearer the middle; but it is unlikely that any serious error due to the presence of water on this board survived the daily reversals of current and the long course of observation to which the apparatus was subjected.

Was a Stable Condition Reached?

The question how long the Thomson effect apparatus would require in order to reach a practically stable condition of temperature, after the heating at one end and the cooling at the other had begun, was one which presented itself and was considered long before the apparatus was set up. With the assumption of no lateral flow of heat from the bars, calculation, by a well-known use of Fourier's series, showed that two hours would be ample time. But such a calculation could not be decisive for the actual conditions of the experiment, where lateral flow did exist to a considerable extent, and where the thermal condition of the board B (Figure 6) might affect in some measure the thermo-electric currents passing through the wires lying upon it. The usual practise in a day's work was to have the thermal conditions at the ends of the apparatus established at least two hours, and the electric current in the bars at least one hour, before observations on the Thomson effect were begun; but it was not taken for granted that a stable condition was reached by this preliminary delay. On the contrary, of the six cycles, or runs, made, as a rule, each day (see p. 31), the first cycle and the first cycle after reversal of the current through the bars, which latter cycle usually began about one hour after the reversal, were at first regarded with much suspicion. Accordingly, the mean result obtained by a rather summary calculation from these suspected cycles in fourteen * days of work was compared with the mean result obtained, in a similar way, from all the cycles of these fourteen days. The difference of the two means was found to be less than one third of one per cent of either mean, and therefore all cycles were treated as entitled to equal confidence.

Nevertheless, observations made with the express purpose of getting data for an estimation of the lateral flow of heat from the apparatus showed that perceptible progressive changes of gradient were going on in the bars after several hours of fixed terminal condition. These changes were probably not of a character to effect perceptibly the differential of gradients, with which we are mainly concerned.

The Lateral Loss of Heat.

This loss has already been referred to, and the general method by which it was estimated has been given on pp. 34 and 35; but some details

* There were fifteen such days of work; but the data of the first day were not sufficiently full to be used here.

of the methods used in getting the data needed in making this estimate may be desirable.

On November 23, with boiling water in both boxes of the Thomson effect apparatus from 12.45 o' clock on, observations made first with the couples K and M (see Figure 3) joined in series, then with the junctions N and P joined in series, then with K and M once more, and so on alternately, indicated gradients of temperature proportional, approximately, to the numbers given below:

Time	K-M		N-P	
3.11	165			
3.24			169	
3.33	151			
4.10	134			
4.21			129	
4.29	125			
4.58	121			
5.08			112	
5.15	114	116		110
5.23			109	
5.31	114			

It is evident that an approximately stable condition had been reached during the last hour of the observations; and the last three values found from K–M, with the last two found from N–P, were taken as indicative of the temperature gradient in this stable condition. The average value of this gradient at the four ends, as estimated from these observations, was 0.104° per centimeter. A corresponding set of observations made December 8 gave 0.108° per centimeter as the mean gradient at the four ends.

As the gradient at the middle of the bars must have been practically zero, we cannot err greatly in taking the mean gradient from the ends to the middle of each 20 cm. cylindrical part as 0.06° per centimeter, and the temperature at the middle as 0.6° lower than that at the ends of this part. This estimate is in fair agreement with the result of a rough direct measurement of this difference made October 29. The mean temperature of that part of bar α which lies between the mid point of KK' and the mid point of MM' must have been not far from 0.3° lower than the temperature of the water at the ends; and a like estimate holds for bar β.

On December 8 an attempt was made to find the temperatures at various points on the guard-ring bars, by means of thermo-electric couples X, Y,

Z, Z', Y' and X'. One junction of each of these couples was attached, with a thin layer of sheet rubber between, to one of the guard-ring bars. Thus, one junction in the case of couple Z (see Figure 4) was at the middle of bar 3; Y was on the same bar, 4.25 cm. from the middle toward the left end; X on the same bar 4.25 cm. beyond Y. Z' was on bar 3 at the middle; Y was on the same bar 4.25 cm. from the middle toward the right end; X' on the same bar 4.25 cm. beyond Y'. * The placing of these junctions was not very exact. The outer junction of each couple was placed in oil in a glass tube about 10 cm. long and perhaps 0.4 cm. in diameter of bore. These tubes were placed in a mixture of ice and water. The couples X, Y, Z, etc., had not been directly calibrated; but similar junctions, A', B', C', and D', made at the same time from the same wire treated in the same way, had been tested for the purpose of getting the reduction factors to be used with X, Y, Z, etc. The use of these factors, applied to the observations made December 8, gave as the temperature of the middle points of the guard-ring bars, 95.1° C.; as the temperature of points 4.25 cm. nearer the ends, 96.0°; as the temperature of points 4.25 cm. nearer still to the ends, 98.3°. It was evident, however, that no great confidence could be placed in the exactness of these numbers; for, with nearly a hundred degrees difference of temperature between the junctions of a couple, an error of one per cent in the reduction factor would make an error of nearly one degree in the estimation of the temperature of the hot junction. Accordingly, on December 22 a new set of observations was made with couples X, Y, Z, etc., the oil tubes containing the outer junctions being in this case placed in steam over boiling water. The temperature of this steam, as found from the barometric pressure, was 100.3° C. The temperature found for the X point of the guard-ring bar 8 was 97.8°; for the Y point of the same bar, 95.4°; for the Z point, 94.2°. For the X' point of the bar 3 the temperature found was 96.5°; for the Y' point, 94.7°; for the Z' point, 93.5°. Thus, the mean temperature indicated for bar 3 was about 0.9° lower than the mean temperature indicated for bar 8. This is hardly surprising, as these two bars lie a quarter circle apart in the guard-ring, and the region of the lower one, No. 8, is somewhat more shielded than the region of the upper bar. More-

* The intention was that junction X should be just opposite, just as far from the adjacent end vessel as the mid points of the couples $K K'$ and $N N'$, and that X' should have a like relation to $M M'$ and $P P'$. All the junctions on α and β were entirely hidden when X, Y, Z, etc., were placed, and the mid points of $K K'$ and $M M'$ were supposed to be 17 cm. apart instead of 16 cm.

over, the ends of the uppermost guard-ring bars were not all the time fully covered by the water in the end vessels. In fact, these two bars were chosen to carry the junctions X, Y, Z, etc., for the reason that they seemed fairly representative, one of conditions in the upper half of the guard-ring, the other of conditions in the lower half.

The observations of December 22 indicate that the mean temperature of the guard-ring bars, for the 17 cm. length extending from the vertical plane of the point X to the plane of the point X', was at the time of these observations about 5° below the temperature of the boiling water in which the ends of the bars were immersed; but a close examination of the observations shows that the apparatus as a whole had not yet reached a perfectly stable condition of temperature. The thermo-electric observations began when the water at the ends of the bars had been boiling not much more than two hours and ended about two hours later; and the earlier observations of the series showed a slightly lower temperature for the bars than the later ones. It is not unlikely that, if the experiment had continued some hours longer, the mean difference of temperature between the boiling water and that part of the guard-ring which lies between the planes of X and X' would have fallen as low as 4.5°; and this is taken as the final value. See p. 35.

On December 9, 1904, with all the usual conditions of an experiment on the Thomson effect, observations were made for the purpose of determining, approximately, the temperatures of the mid points of the couples KK', NN', etc., on the bars α and β. The outer junctions belonging to E, F, G, etc., were placed in oil tubes, in ice and water, and after the terminal thermal conditions of the main apparatus had been established, and the main electric current had been running nearly eight hours, observations on the current from the couples J and G., joined in series with each other, were made. They were followed by observations on couples I and F, joined in series; and so on. The general course of these observations was very satisfactory, and it indicated the following values:

Mean temperatures of the points	J	and	G	$=$	90.2°
"	"	"	I	"	$F = 51.0°$
"	"	"	H	"	$E = 13.0°$

On the same day, and after the observations just described, similar observations were made with the couples X, Y, Z, etc., taken singly. The following values were indicated:

Temperatures of the point $X' = 88.7°$
" " $Y' = 67.8°$
" " $Z' = 49.0°$
" " $Z = 49.4°$
" " $Y = 33.2°$
" " $X = 12.0°$

As the junctions X, Y, etc., were not very accurately placed on their respective bars, it would have been desirable to make a corresponding set of observations on these junctions with the X end hot and the X' end cold, if there had been any need of using these last guard-ring temperatures in the calculation of the Thomson effect. That there was no such need is shown by the considerations given on p. 35.

Temperature Coefficient of Thermal Conductivity.

The measurements designed especially for the determination of this coefficient (see p. 33) were made in February, 1905, after the observations on the Thomson effect were ended. These especial measurements involved few, if any, considerations that have not been set forth in the preceding pages.

With no electric current in α and β, but with a permanent gradient of temperature established therein, let

g_h = gradient of temperature at the hot end,
g_c = " " " " " cold " ,
t_h = temperature of mid point of hot end couple,
t_c = " " " " " cold " " ,
k_h = thermal conductivity at temperature t_h,
k_c = " " " " t_c,
f = temperature coefficient of thermal conductivity.

Then, if there were no lateral flow of heat from α and β, we should have, neglecting the small effect of expansion of the iron,

$$k_h \times g_h = k_c \times g_c, \quad \text{or} \quad \frac{k_h}{k_c} = \frac{g_c}{g_h}, \quad \text{and so}$$

$$(8) \quad f = \frac{k_h - k_c}{k_c} \div (t_h - t_c) = \left(\frac{k_h}{k_c} - 1\right) \div (t_h - t_c) = \left(\frac{g_c}{g_h} - 1\right) \div (t_h - t_c).$$

Observations on the ratio $g_c \div g_h$ were made on four days, with the following results:

	$g_c \div g_h$	t_c	t_h	
Feb. 1	0.9312	13.0°	[87.3°]	Right end hot
" 4	0.9320		[87.5°]	" " "
" 7	0.9478	12.0°	87.3°	" " cold
" 9	0.9460	12.7°	[87.2°]	" " "
Means	0.9392	12.6°	87.3°	

The marked difference between the first two and the last two values found for $g_c \div g_h$ is due to the reversal of the direction of flow, there being, no doubt, small differences in the placing of the thermo-electric couples at the ends of the bars. See p. 27. The reversal should eliminate error due to this source.

The value 87.3°, found for t_h on February 7, was obtained from observations made with thermo-electric couple H, the outer junction of this couple being placed, on this day, in oil heated to a temperature not far from that of the inner junction, so that uncertainty of calibration of the couple had little effect on the result. The other values of t_h, which are given in brackets, were estimated from this value, with use of the barometer observations of the various days as an indication of the various temperatures of the boiling water.

Estimation of the loss of heat by lateral flow required observations on the difference of temperature between points on the main bars, with no electric current on, and corresponding points on the guard-ring bars. Such observations were obstructed, and in some measure restricted, by the breaking of one of the thermo-electric wires and by the inadequate length of several others, the full scope of their possible use not having been foreseen when the couples were made. The results were as follows:

	On α and β.			On guard-ring.		
	t_c	t_m	t_h	t_c	t_m	t_h
Feb. 1	13.0° [*E–H.*]					
" 4		49.3° [*I*]			48.6° [*Z*] 47.8° [*Z'*]	87.1° [*X'*]
" 7	12.0° [*G*]	48.7° [*I*]	87.3° [*H*]	11.3° [*X'*]	47.4° [*Z'*] 48.0° [*Z*]	86.7° [*X*]
" 9	12 7° [*G*]			12.1° [*X'*]		
	12.6°	49.0°	87.3°	11.7°	48.0°	86.9°

Taking account now of the fact that the junctions X and X' were not placed just opposite H and G (see p. 50), and estimating for points which are opposite H and G, we get

On α and β.			On guard-ring.		
t_c	t_m	t_h	t_c	t_m	t_h
12.6°	49.0°	87.3°	14.0°	48.0°	84.6°

Accordingly we get 0.8° as an approximate value for the mean excess of temperature of the bars α and β, for the significant 16 cm. between mid points of hot and cold couples, above the temperature of the corresponding part of the guard-ring bars. Now from p. 35 it appears that the lateral flow from the same 16 cm. of each bar, when it is 4.2° warmer than the guard-ring, is 0.0242 calorie per second. With a difference of 0.8° we may take the lateral flow from each bar as 0.0046 calorie per second, about 0.9 per cent of the amount which is conducted along the hot end of the bar by a gradient of 4.6° per centimeter.

Making correction accordingly, and taking account also of the slight effect produced by the expansion of the main bars, we get from the value 0.9392, given above for $g_c \div g_h$, the ratio $k_h \div k_c = 0.947$; and hence

$$f = \left(\frac{k_h}{k_c} - 1\right) \div (t_h - t_c) = (0.947 - 1) \div (87.3 - 12.6) = -0.0007.$$

This study of the value of f raised again the question of the accuracy of calibration of the thermo-electric couples; and a good deal of additional attention was given to this matter in February, as the following summary will show:

	Thermo-electric Height (mean) for A', B', C', D.	Mean t.	Range of t.
Feb. 14	2056×10^{-8}	89.3°	78.5° to 100.2°
" 16 and 17	1697×10^{-8}	17.3°	0° to 34.6°
	A, B, C, D.		
" 20 and 21	2054×10^{-8}	89.5°	78.6° 100.4°
" 23	1709×10^{-8}	17.4°	0° 34.8°

Combination gives

$$2055 \times 10^{-8} \text{ at } 89.4°,$$
$$1703 \times 10^{-8} \text{ at } 17.4°.$$

Mr. Churchill found values from which we get

$$2070 \times 10^{-8} \text{ at } 89.4°,$$
$$1722 \times 10^{-8} \text{ at } 17.4°.$$

Ratios of Churchill values to February values

$$2070 \div 2055 = 1.0073 \text{ at } 89.4°,$$
$$1722 \div 1703 = 1.0111 \text{ at } 17.3°.$$

Change from one set of calibration values to the other would make but little difference in the value found for the Thomson effect coefficient and no large difference in the value found for f. Accordingly, the thermo-electric heights found by Mr. Churchill have been retained in all the important calculations of this paper.